Astronomy 680: Notes on CCD S/N

Reminder: For CCDs, the gain $g \equiv$(input signal / output signal) or $g = input\ (e^-)\ /\ output(ADU)$. This is the inverse of how the term is used in most other circumstances.

Signal–to–Noise Ratio (S/N):

Caution: Never try to calculate S/N in ADU (or "counts"). Always convert to photons (or electrons) first.

The signal–to–noise ratio (S/N) is a very useful tool for quantifying the quality of an observation, for determining the predicted quality of a planned observation, and for comparing different telescope+detector configurations. For example, a S/N = 10 tells us immediately that the data are good to ∼10%, and a S/N = 100 tells us the data are good to ∼1%.

To compute the S/N, let $N_* =$ the total number of photons (or e^-) detected from the target or source (e.g. the star of interest): $N_* = \sum_{i=1}^{npix} f_i$, where npix is the number of pixels inside the user–defined soft aperture. N_* is the "signal".

The "noise" comes from several sources and these must be added together in quadrature following the usual propagation of errors method: $\sigma_{noise} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 ...}$ Let
$n_s =$ number of background sky photons *per pixel*; so $npix \times n_s = N_s$ the total sky photons;
$\sigma_{RO} =$ readout noise in e^- *per pixel*.
And for use later, let's define:
$g =$ average detector gain (in e^- / ADU);
$q =$ average detector quantum efficiency (Q.E.);
$t =$ integration time in seconds (i.e. exposure time).

The detector dark noise N_{dark} is being ignored since it contributes a negligible amount of noise. This is true for LN2 cooled CCDs in the optical but definitely not true when working in the IR (or with "amateur astronomy grade" CCDs, e.g. SBIG's *ST* line of CCDs). In general, dark noise depends upon the exposure time and follows Poisson statistics. Scintillation noise (which is directly proportional to the source brightness) is also being ignored. For very bright stars ($< 8\ mag$) and short exposures (< few seconds) on a small telescope ($< 1m$), scintillation noise can dominate!

Using the above definitions, the S/N ratio is given by the expression:

$$S/N = \frac{N_*}{\sqrt{N_* + npix(n_s + \sigma_{RO}^2)}} \tag{1}$$

The first term in the noise is due to the source itself: $\sigma_{N_*}^2 = N_*$ from Poisson statistics. The next term is the sky noise and the final term is the readout noise. Note that the readout noise enters as σ^2, not σ, since it is does not follow Poisson statistics. Let's look at some limiting cases:

(1) If the source dominates (i.e. the source is very bright), then $N_* >> npix(n_s + \sigma_{RO}^2)$ and the S/N is $\approx \sqrt{N_*}$. This is simply Poisson statistics. This approximation is very easy and quick to compute so it is quite useful as a "back of the envelope" estimation. For example, if you need to have a precision of better than 1% you need to be collecting > 10,000 photons; for better than 0.1% you need $> 10^6$ photons. However, don't forget that this is the best S/N obtainable — a true upper limit — and actual performance will *always* be worse. Notice that since S/N $\approx \sqrt{N_*}$ and $N_* \propto t$ (the exposure time), the S/N grows as the $\sqrt{t}$. More on this later.

(2) If the background sky dominates, for example a faint target on a moonlit night, then the $S/N \approx \frac{N_*}{\sqrt{npix\ n_s}} = \frac{N_*}{\sqrt{N_s}}$. The S/N is again $\propto \sqrt{t}$ so you only gain a small amount by taking longer exposures.

(3) If the detector readout noise dominates, for example a very short exposure or a very faint star and sky, then the $S/N \approx \frac{N_*}{\sigma_{RO} \times \sqrt{npix}}$. Since σ_{RO} is independent of exposure time, the $S/N \propto t$: you gain a lot by lengthening the integration times.

Remember that all of the above S/N relations use electrons, not ADU. But at the telescope you measure ADU from the CCD, not e^-. How does one determine the S/N when using measurements that come in units of ADU? The conversion factor is the gain g: $N_{*_{e^-}} = N_{*_{ADU}} \times g$. Likewise for the sky (and dark) noise. So if you work with ADU, then you can re-cast equation (1) as:

$$S/N = \frac{N_{*_{ADU}}\ g}{\sqrt{N_{*_{ADU}}\ g + npix(N_{s_{ADU}}\ g + \sigma_{RO}^2)}}. \qquad (2)$$

Notice that since the readout noise is in fact always given in e^-/pix, not ADU/pix, it is much better to leave this very last term as σ_{RO}^2 not $(\sigma_{RO_{ADU}}^2 g^2)$.

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Now let's expand the various S/N terms into more fundamental units, and use ADU whenever possible. The source signal N_* in photons can be written as $N_* = \dot{N}_* \times t \times q$, where $\dot{N}_*$ is the photon arrival rate, t is the exposure time, and q is the detector quantum efficiency. $\dot{N}_*$ depends on the collecting area of the telescope ($\propto diameter^2$), the filters, reflectivity of the mirrors, transparency of the atmosphere, airmass, etc.. So then, using count rates in units of e^-/s (or $photons/s$), we can re-write equation (1) as:

$$S/N = \frac{\dot{N}_* tq}{\sqrt{\dot{N}_* tq + npix(\dot{n}_s tq + \sigma_{RO}^2)}} \quad , \qquad (3)$$

If we use the measured count rate in ADU then the signal is: $N_* = (\dot{N}_{*_{ADU}} \times g) \times t \times q$. The signal N_* is still in units of e^-, but the photon rate ($\dot{N}_{*_{ADU}}$) is now given in the measured value of ADU/s and so must be multiplied by the gain. The noise in e^- is: $\sqrt{\dot{N}_{*_{ADU}} gtq + npix(\dot{n}_{s_{ADU}} gtq + \sigma_{RO}^2)}$.

So using count rates in ADU/s (since ADU are what is actually recorded in the fits files) equation (3) becomes:

$$S/N = \frac{\dot{N}_{*_{ADU}} gtq}{\sqrt{\dot{N}_{*_{ADU}} gtq + npix(\dot{n}_{s_{ADU}} gtq + \sigma_{RO}^2)}}. \tag{4}$$

We can easily recast expressions (3) or (4) as quadratic functions of t. Then we can solve for t for a given S/N. In other words, we can calculate how long it will take to reach a desired S/N level: $t = f(\ S/N; \dot{N}_*, \dot{n}_s, npix, \sigma_{RO}^2, g, q)$. Notice that through the $\dot{N}_*$ term, the time t to reach a given S/N will depend on the size of the telescope and the throughput of the atmosphere + filters + instrument optics, etc..

Finally, notice that since *npix* is related to the seeing ($npix \propto seeing^2$), the better the seeing, the smaller *npix*, and the better the S/N. This is especially important in cases where the background or detector noise dominates (e.g. very faint sources). So the lesson is this: Do whatever you can to keep the seeing as crisp as possible. For examples, keep the telescope cold; make sure the telescope is guiding as best it can (adjust the tracking rate and/or make sure the telescope is balanced properly); carefully focus the telescope; and if the temperature changes (and hence the telescope suffers thermal expansion), adjust the mirror position so the focus is as good as possible. It will be worth it!